1. Calculate the energy of the photon, given the frequency ($\nu = 2\times10^{9}$):

$$E = h\,\nu = 4.136\times10^{-15}\bullet 2\times10^{9} = 8.27\times10^{-6}\text{ eV}$$

$$= 6.626\times10^{-34}\bullet 2\times10^{9} = 1.325\times10^{-24}\text{ Joules}$$

2. What is the frequency of a 1 mm wave?

$$\lambda f = c;\quad f = \frac{3\times10^{8}}{1\times10^{-3}} = 3\times10^{11}\text{ Hz}$$

3. Black - body radiation problem:

$$T = 20 + 273 = 293\text{ K}$$

$$\text{Power / area} = \sigma T^{4} = 5.67\times10^{-8}\bullet 293^{4} = 417.9\text{ Watts / m}^{2}$$

$$\text{Power} = 4\pi r^{2}\bullet 417.9 = 5.25\text{ kiloWatts}$$

4. At what wavelength does the radiation peak?

For T = 20 C = 293 K

$$\lambda_{max} = \frac{a}{T} = \frac{2.898\times10^{-3}}{293} = 9.89\times10^{-6}\text{ m}$$

For T = 1000 C = 1273 K

$$\lambda_{max} = \frac{a}{T} = \frac{2.898\times10^{-3}}{1273} = 2.277\times10^{-6}\text{ m}$$

5. See below

6. For a Bohr atom, find the energy for the n =3 to n = 2 transition.

$$\Delta E = E_f - E_i = -13.58\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = -13.58\left(\frac{1}{4} - \frac{1}{9}\right) = -1.886\text{ eV}$$

The electron loses 1.9 eV energy. That energy goes to the photon.

$$E = h\,f = 4.136\times10^{-15}\bullet f = 1.886\text{ eV};\quad f = 4.56\times10^{14}$$

$$\lambda f = c;\quad \lambda = \frac{3\times10^{8}}{4.56\times10^{14}} = 6.58\times10^{-7}\text{ meters}$$

This is termed the H-alpha (H_α) transition, and it is visible (red light)

7. There is a small problem of units, here. I choose to convert temperature to Joules, then eV....

$$kT = 1.38\times10^{-23}\bullet 1000 = 1.38\times10^{-20}\text{ J}$$

$$kT = 1.38\times10^{-20}/1.6\times10^{-19} = 8.63\times10^{-2}\text{ eV}$$

The average kinetic energy of one atom is then given by:

$$\tfrac{1}{2}mv^{2} = \frac{3}{2}kT = 1.5\bullet 8.63\times10^{-2} = 0.13\text{ eV}.$$

Note that the factor of 1.5 is really a detail....We take the atom to be hydrogen

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \bullet 1.6 \times 10^{-19} \bullet 0.13}{1.67 \times 10^{-27}}} = 5.0 \times 10^{3} \ \ m/s$$

what would the velocity be for oxygen? (mass = 16* 1.67×10^{-27})

9. Debye Shielding

a) T = 1,000 K, n = 1×10^5 electrons/cm^3.

$$\lambda_D \ (\text{meters}) = \left(\frac{\varepsilon_0 \ kT}{e^2 \ n_e}\right)^{1/2} = \left(\frac{8.854 \times 10^{-12} \bullet 1.38 \times 10^{-23} \ \frac{Joules}{Kelvin} \bullet 1000K}{\left(1.6 \times 10^{-19}\right)^2 \bullet \left(10^5 \frac{e^-}{cm^3} \bullet 10^6 \frac{cm^3}{m^3}\right)}\right)^{\frac{1}{2}}$$

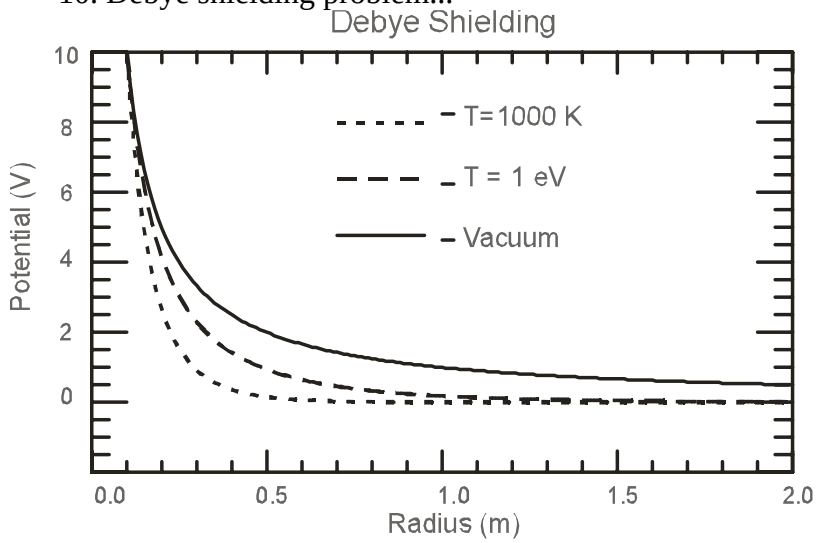
= 0.0069 m = 6.9 mm

b) T = 500 eV, n = 1 electron/cm^3

$$\lambda_D \ (\text{meters}) = \left(\frac{\varepsilon_0 \ kT}{e^2 \ n_e}\right)^{1/2} = \left(\frac{8.854 \times 10^{-12} \bullet 1.6 \times 10^{-19} \ \frac{Joules}{eV} \bullet 500 \text{ eV}}{\left(1.6 \times 10^{-19}\right)^2 \bullet \left(1 \frac{e^-}{cm^3} \bullet 10^6 \frac{cm^3}{m^3}\right)}\right)^{\frac{1}{2}}$$

= 166.34 meters

10. Debye shielding problem...



The general formula is: $\lambda_D\ (\text{meters}) = \left(\dfrac{\varepsilon_0\ kT}{e^2\ n_e}\right)^{1/2}$

but it is easier to use the numerical forms:

(A) $n = 2.0\times10^8,\ \ T = 1000\ \text{K};$

$$\lambda_D = 69\left(\frac{T}{n}\right)^{\frac{1}{2}} = 69\left(\frac{1000}{2.0\times10^8}\right)^{\frac{1}{2}} = 0.154\ \text{m; and,}$$

(B) $n = 2.0\times10^8,\ \ T = 1\ \text{eV}\ \ \lambda_D = 7430\left(\dfrac{T}{n}\right)^{\frac{1}{2}} = 7430\left(\dfrac{1}{2.0\times10^8}\right)^{\frac{1}{2}} = 0.525\ \text{m}$

now just plot up $\Phi = \dfrac{1}{4\pi\varepsilon_\circ}\dfrac{q}{r}\ e^{-(r-r_0)/\lambda_D}$

Note that the solid line in the plot is for no Debye sheath (just a coulomb potential)

11. B = 100 nano-Tesla,

$$B = 100 \times 10^{-9}\ \text{T},\quad f = \frac{qB}{2\pi m} = \frac{1.6 \times 10^{-19} \bullet 100 \times 10^{-9}}{2\pi \bullet 1.67 \times 10^{-27}} = 1.525\ \text{Hz}$$

12. Use $B = 4 \times 10^{-5}$ Tesla (the value near Sri Lanka)

$$f = \frac{qB}{2\pi m} = \frac{1.6 \times 10^{-19} \bullet 4 \times 10^{-5}}{2\pi \bullet 9.1 \times 10^{-31}} = 1.12 \times 10^{6}\,\text{Hz} \approx 1\ \text{MHz}$$

13. Drift velocity (E cross B). Note that the drift velocity is independent of energy - the value of 5 eV is extraneous information.

$$E = \frac{\mathbf{E} \times \mathbf{B}}{B^2} = \frac{E}{B} = \frac{10^{-3}}{200 \times 10^{-9}} = 5000 = 5\ \text{km/s}$$

14. Plasma pressure...

$$p = \sum n_i k T_i = 2\left(2.0 \times 10^{10} \bullet 0.5\right) = 2.0 \times 10^{10}\ \text{eV}/\text{m}^3$$

$$= 1.6 \times 10^{-19} \bullet 2.0 \times 10^{10} = 3.2 \times 10^{-9}\ \text{J}/\text{m}^3$$

the factor of '2' comes from having ions and electrons.

The magnetic energy density (presssure) is given by:

$$p = \frac{B^2}{2\mu_\circ} = \frac{\left(3 \times 10^{-6}\right)^2}{2 \bullet 4\pi \times 10^{-7}} = 3.58 \times 10^{-6}\ \text{J}/\text{m}^3$$

The ratio is $\beta = \dfrac{\sum n_i k T_i}{B^2/2\mu_\circ} = \dfrac{3.2 \times 10^{-9}}{3.58 \times 10^{-6}} = 8.94 \times 10^{-4}$, so the magnetic field dominates

Black Body Problem

Calculate the equilibrium temperature of the earth, or any satellite orbiting the sun.

How much energy is radiated by the sun, and how much then reaches earth orbit?

($R_{sun} = 6.96\times10^{8}$ m; $T_{sun} = 5800$ K; $\sigma = 5.67\times10^{-8}\ W/m^2K^4$)

The radiated power is:

$$P_{sun} = \varepsilon\ \sigma\ T_{sun}^4 \bullet 4\pi R_{Sun}^2 = 1\bullet 5.67\times10^{-8}\bullet 5800^4 \bullet 4\pi \bullet \left(6.96\times10^{8}\right)^2$$

= 3.90592e+026 Joules/Second or Watts

Textbook Answer: 3.83×10^{26} Joules/s

The radiation spreads as the electromagnetic energy moves outwards, according to Gauss' Law (conservation of energy, really). What is the radiation at earth's orbit?

$\left(R_{\text{Earth Orbit}} = 150\times10^{9}\ \text{m, aka 93 million miles}\right)$

$$R = P_{solar}\Big/\left(4\pi\ R_{\text{Earth Orbit}}^2\right) = \varepsilon\ \sigma\ T_{sun}^4 \bullet \left(\frac{R_{Sun}}{R_{\text{Earth Orbit}}}\right)^2 = 1\bullet 5.67\times10^{-8}\bullet 5800^4 \bullet \left(\frac{6.96\times10^{8}}{1.5\times10^{11}}\right)^2$$

$= 1381.44\ \text{Watts/m}^2$

which in fact is real, real close to the correct value.

How much power does the earth receive?

The radiation is incident on a cross-section area which is the area of the earth's disk: $\pi\ R_{Earth}^2$. $\left(R_{Earth} = 6380\ \text{km}\right)$. One question which can be left open at this point is the question of the earth's albedo - the factor which determines what fraction of the sunlight incident on the earth is absorbed, vs reflected. Here, we will use the symbol "α" for albedo.

$$P_{Solar} = \alpha\ \sigma\ T_{Sun}^4 \bullet \left(\frac{R_{Sun}}{R_{\text{Earth Orbit}}}\right)^2 \bullet \pi\ R_{Earth}^2 = \alpha \bullet 1.77\times10^{17}\ \text{Watts}$$

where you can nominally take α to be one for now.

This power has to be re-radiated by the earth - once a steady state temperature has been reached. Hence, you can derive a temperature for the earth.

$$P_{Solar} = \alpha\ \sigma\ T_{Sun}^4 \bullet \left(\frac{R_{Sun}}{R_{Earth\ Orbit}}\right)^2 \bullet \pi\ R_{Earth}^2 = \varepsilon_{Earth}\ \sigma\ T_{Earth}^4 \bullet 4\pi\ R_{Earth}^2$$

$$\alpha\ T_{Sun}^4 \bullet \left(\frac{R_{Sun}}{R_{Earth\ Orbit}}\right)^2 = 4\varepsilon_{Earth}\ T_{Earth}^4$$

$$T_{Earth}^4 = \frac{\alpha}{4\varepsilon_{Earth}} \left(\frac{R_{Sun}}{R_{Earth\ Orbit}}\right)^2 \bullet T_{Sun}^4$$

$$T_{Earth} = \left(\frac{\alpha}{\varepsilon_{Earth}}\right)^{\frac{1}{4}} \left(\frac{R_{Sun}}{2\ R_{Earth\ Orbit}}\right)^{\frac{1}{2}} \bullet T_{Sun} = \left(\frac{\alpha}{\varepsilon_{Earth}}\right)^{\frac{1}{4}} \bullet 280\ K$$

The earth will freeze if the number drops to 273 K. If the emissivity of the earth is one, what does α have to drop to for this to happen?

$$T_{Earth} = 273 = \left(\frac{\alpha}{\varepsilon_{Earth}}\right)^{\frac{1}{4}} \bullet 280\ K$$

$$\alpha = \left(\frac{273}{280}\right)^4 = 0.904$$