1. n = 10 cm^{-3} = 10^7 m^{-3}; v = 400 km/s = 4×10^5 m/s; m = 1.67×10^{-27} kg; B = 7 nT = 7 $\times 10^{-9}$ Tesla.

$$\beta = \frac{n\left(\frac{1}{2}\ mv^2\right)}{B^2/2\mu_o} = \frac{10^7 \bullet 0.5 \bullet 1.67 \times 10^{-27} \bullet 16 \times 10^{10}}{49 \times 10^{-18}/8\pi \times 10^{-7}} = \frac{1.34 \times 10^{-9}}{1.95 \times 10^{-11}} = 68.5$$

- the plasma pressure dominates

2. not without knowing the temperature of the star

3. $\tan \Psi = \frac{\omega r}{v}$ is the formula. For 400 km/s solar wind, the RHS equals 1, so ψ = 45°. Double v, tan Ψ drops to 0.5, Ψ drops to 26.6°

5. Average kinetic energy is: $\frac{1}{2} m v_{thermal}^2 = \frac{3}{2} kT$, $v_{thermal} = \sqrt{3kT/m}$

ions: $v_{thermal} = \sqrt{3 \bullet 1.6 \times 10^{-19} \bullet 7 / 1.67 \times 10^{-27}}$ $= \sqrt{2.01 \times 10^9} = 4.5 \times 10^4\ \frac{m}{s} = 45 \frac{km}{s}$	electrons: $v_{thermal} = \sqrt{3 \bullet 1.6 \times 10^{-19} \bullet 10 / 9.1 \times 10^{-31}}$ $= \sqrt{5.27 \times 10^{12}} = 2.3 \times 10^6\ \frac{m}{s} = 2300 \frac{km}{s}$
Mach Number = $\frac{v_{wind}}{v_{thermal}} = \frac{400 \frac{km}{s}}{45 \frac{km}{s}} = 8.9$	Mach Number = $\frac{v_{wind}}{v_{thermal}} = \frac{400 \frac{km}{s}}{2300 \frac{km}{s}} = 0.17$

Problem 7.

The assignment solution ought to look something like the following, for the given parameters. The point is that the product of the exponentially decaying barometric relation, times the area of the surface (r^2), can produce a velocity which is increasing, and can increase to a Mach number above 1. The second figure shows a family of curves, normalized by the appropriate Mach number for each temperature. The primary limitation in this simple look at the acceleration of the solar wind is the neglect of the effect of the plasma pressure on the expansion.

$$n(r) = n_o \exp\left[\frac{G\, M_\circ m_H}{kT\ R_\circ}\left(\frac{1}{r} - \frac{1}{r_0}\right)\right]$$

$m_H = 1.67 \times 10^{-27}$; $G = 6.67 \times 10^{-11}$ Newton-m^2/kg^2

$n_o = \rho_o/m_H = \frac{10^{-7}\ \frac{kg}{m^3}}{1.67 \times 10^{-27}} = 6 \times 10^{19}\ \frac{protons}{m^3}$; r_o = 1 solar radius;

T is 1.0×10^6 - 2.0×10^6; $k = 1.38 \times 10^{-23}$ J/K

$M_o = 1.989\times10^{30}$ kg; $R_o = 6.96\times10^8$ m

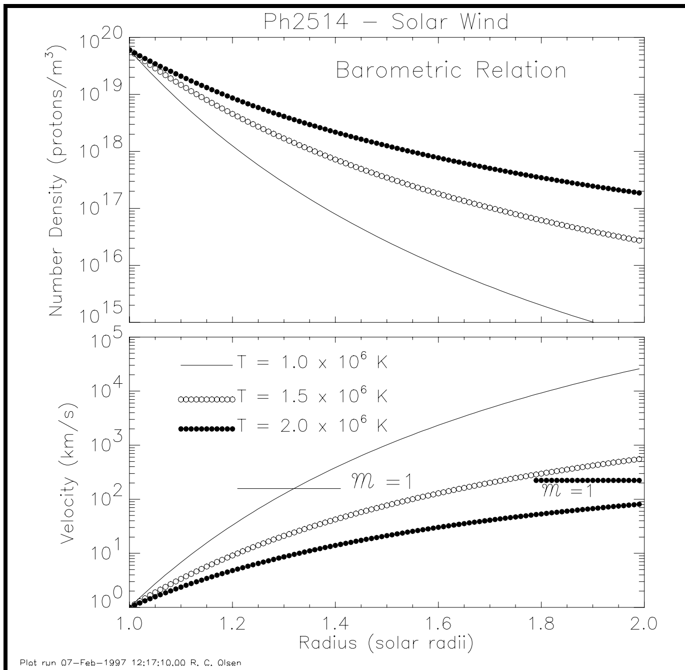
$$n(r) = n_o \exp\left[\frac{23.07}{T(\text{millions of degrees})}\left(\frac{1}{r} - \frac{1}{r_o}\right)\right]$$

Flux through the (Gaussian) surface $(n \bullet v \bullet \text{Area})$ is the conserved quantity:

$n_o \bullet v_o \bullet r_o^2 = n \bullet v \bullet r^2$. Taking $r_o = 1$

$$n_o v_o = n_o \exp\left[\frac{G M_\odot m_H}{kT R_\odot}\left(\frac{1}{r} - \frac{1}{r_o}\right)\right] v r^2$$

$$\frac{v}{v_o} = \frac{1}{r^2}\exp\left[\frac{23.07}{T}\left(1 - \frac{1}{r}\right)\right]$$



One question you might ask is what happens beyond r = 2 solar radii in this model. The answer is modified slightly - you can see you really have to go up to 2.25 million degrees before the solar wind will stabilize at a subsonic value.