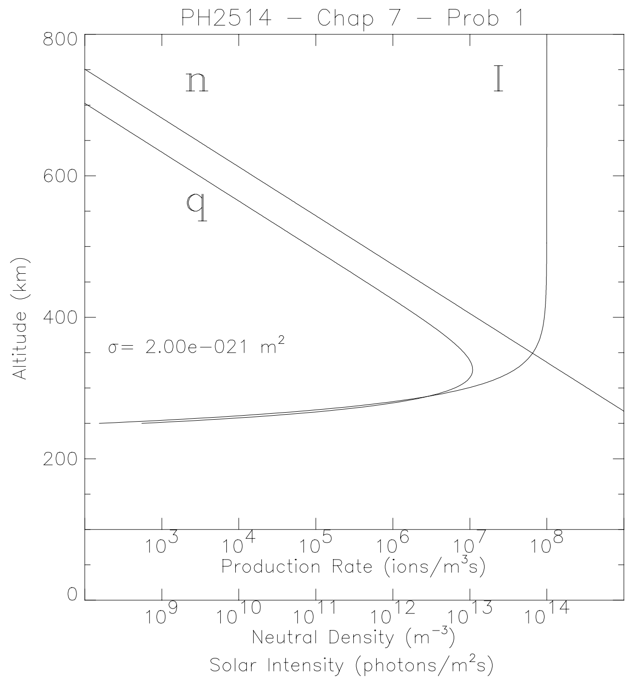
1. Determine the production curve for a Chapman layer, using the formula:

$q(h) = I(h)\, \sigma\, n(h)$
electrons/m^3 s

$n = n_o e^{-\frac{h-h_o}{H}}$; H=30 km;
$n_o = 5 \times 10^{16}$ molecules/m^{-3} ,
$h_o = 150$ km

$I(h) = I_o\, e^{-(1000 - h)/h_o} =$
intensity of the solar radiation (in photons/m^2s) at the altitude h;
$I_o = 1.0 \times 10^{14}$,
$h_o(\mathrm{km}) = 10^{17}/n$.

$\sigma = 2 \times 10^{-21}$



7-2. Figure 7.12 shows an electron density profile. Estimate the scale height for plasma above 700 km altitude. Use this scale height to estimate the temperature. Note that the answer depends upon which mass you use in the scale height formula. Try both electron mass and atomic oxygen (16 amu). Which is more reasonable?

Scale height - use same approach as in chapter 6 homework:
Looking at the figure, I get:
$n = 2{\times}10^{11}$ m^{-3} at h = 700, and
$n = 4{\times}10^{9}$ m^{-3} at h = 5000, and

$$n = n_o\, e^{-\frac{(h-700)}{H}} \Rightarrow 4\times10^9 = 2\times10^{11}\, e^{-\frac{(5000-700)}{H}}$$

$$e^{\frac{(5000-700)}{H}} = 50 \text{ or } H = \frac{4300}{\ln(50)} = 1.1\times10^3$$

The scale height is over 1000 km

$$H_p = \frac{kT}{\mu\, m_p g} \Rightarrow T = \frac{H\,\mu\, m\, g}{k}$$

Begin by assuming that the mass is from the ions, so $\mu = 16$, $m = 1.67{\times}10^{-27}$

$$T = \frac{1.1\times10^6 \bullet 16 \bullet 1.67\times10^{-27} \bullet 9.8}{1.38\times10^{-23}} = 2.1\times10^4 \text{ Kelvin}$$

This seems a little high. Let's see what happens for an electron mass:

$$T = \frac{1.1\times10^6 \bullet 9.1\times10^{-31} \bullet 9.8}{1.38\times10^{-23}} = 0.7 \text{ Kelvin}$$

That's worse….
The scale height is probably too large by a factor of 2-5, but the primary problem is the substantial contribution of H+ to the ionospheric mixture. Why doesn't the electron mass determine the electron scale height? (Sort of the point of the problem.) The answer is that the upward (random) motion of the electrons is restrained by the heavier ions (like an anchor). Hence, the ion mass is what determines the ionospheric scale height, to the extent that these plasmas obey a barometric relation.

7-3 From Figure 7.12 estimate the electron density at 200 km and 400 km. What are the radio frequencies that will be reflected at these altitudes assuming vertically traveling waves?

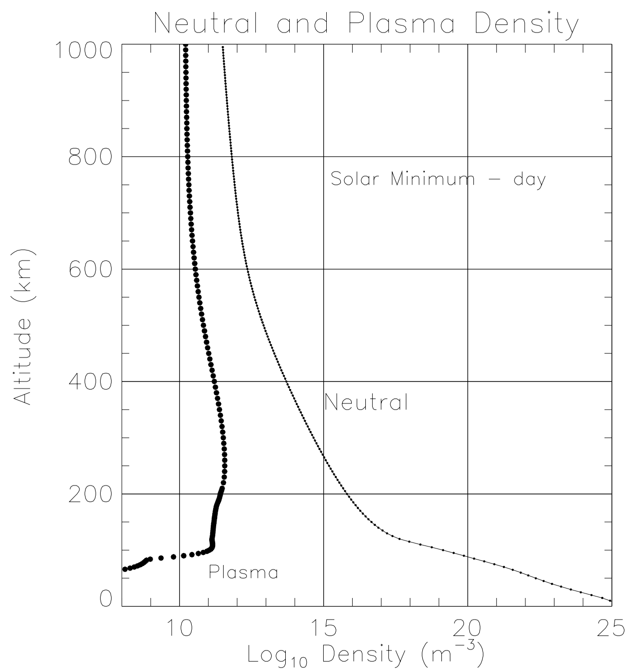
h = 200 km; $n = 2\times10^{11}\ \mathrm{m}^{-3}$

$\Rightarrow\ 4.0\times10^{6}$ Hz

h = 400 km; $n = 9\times10^{11}\ \mathrm{m}^{-3}$

$\Rightarrow\ 8.5\times10^{6}$ Hz

7-4. Compare the neutral gas density and plasma (electron) density as a function of altitude. Plot the total neutral density (number/volume) as a function of altitude (e.g. figure 6.5b). On the same curve, plot the electron density.



7-5. Using the data found in Figure 7.11, estimate the electron density profile. (θ = 0)

7-6 The atmosphere of a hypothetical planet is composed of nitrogen, N_2, and oxygen, O_2. Above 150 km, the atmosphere is isothermal at a temperature of 1500 K.

The number densities of N_2 and O_2 are 5×10^8 cm^{-3} and 5×10^7 cm^{-3} respectively at 200 km; and their **absorption** cross-sections for solar radiation of wavelength 75 nano-meters are 3.1×10^{-17} cm^2 and 2.9×10^{-17} cm^2 respectively.

The **ionisation** cross-section of N_2 for the same radiation is 2.3×10^{-17} cm^2 .

If the solar irradiance at the top of the atmosphere is 2.5×10^9 photons/cm^2 s at 75 nm, what would be the irradiance at 200 km when the radiation is vertically incident on the atmosphere?

$$q(200\text{ km}) = \gamma\, I(200\text{ km})\, n_{N_2}(200\text{ km})\, \sigma_{\text{ionization}}$$

$$I = I_o\, e^{-n_{200}\sigma H \sec\chi}; \quad H = \frac{kT}{mg}; \ \chi = 0$$

Because there are two species of absorbing molecules, the intensity drops due to the sum of the absorptions..

$$I = I_o\, e^{-\left(n_{N_2}\sigma_{\text{absorp } N_2} H_{N_2} + n_{O_2}\sigma_{\text{absorp } O_2} H_{O_2}\right)}$$

$$H_{N_2} = \frac{kT}{mg} = \frac{1.38\times 10^{-23} \bullet 1500}{28 \bullet 1.67\times 10^{-27} \bullet 9.81} = 45{,}100 \text{ m}; \quad 45.1 \text{ km}$$

$$H_{O_2} = \frac{kT}{mg} = \frac{1.38\times 10^{-23} \bullet 1500}{32 \bullet 1.67\times 10^{-27} \bullet 9.81} = 39{,}500 \text{ m}; \quad 39.5 \text{ km}$$

$$n_{N_2}\sigma_{\text{absorp } N_2} H_{N_2} + n_{O_2}\sigma_{\text{absorp } O_2} H_{O_2}$$

$$= 5 \times 10^8 \text{ cm}^{-3} \bullet\ 3.1 \times 10^{-17} \text{ cm}^2\ H_{N_2} + 5 \times 10^7 \text{ cm}^{-3} \bullet\ 2.9 \times 10^{-17} \text{ cm}^2\ H_{O_2}$$

$$= 5\times 10^{14} \text{ m}^{-3} \bullet\ 3.1\times 10^{-21} \text{ m}^2 \bullet 4.51\times 10^4 + 5\times 10^{13} \text{ m}^{-3} \bullet\ 2.9\times 10^{-21} \text{ m}^2 \bullet 3.95\times 10^4$$

$$= 7.0\times 10^{-2} + 5.7 \times 10^{-3} = 7.6\times 10^{-2}$$

$$I = I_o\, e^{-\left(7.6\times 10^{-2}\right)} = I_o \bullet 0.93 = 0.93 \bullet\ 2.5 \times 10^9 = 2.3 \times 10^9 \text{ photons/cm}^2 \text{ s}$$

$$= 2.3 \times 10^{13} \text{ photons/m}^2 \text{ s}$$

Calculate the ionization rate of N_2 at 200 km due to the 75 nm radiation.

$$q(200\text{ km}) = \gamma\, I(200\text{ km})\, n_{N_2}(200\text{ km})\, \sigma_{\text{ionization}}$$

$$q(200\text{ km}) = \gamma \bullet 2.3 \times 10^{9}\text{ photons}/\text{cm}^2\text{ s} \bullet 5 \times 10^{8}\text{ cm}^{-3} \bullet 2.3 \times 10^{-17}\text{ cm}^{-2}$$

$$= \gamma \quad 26.7\ \text{ions}/_{\text{cm}^3\text{ s}} = \gamma \quad 26.7 \times 10^{6}\ \text{ions}/_{\text{m}^3\text{ s}}$$

Take $\gamma=1$

Assuming that the only loss process for N_2^+ is charge transfer to O_2, and that this reaction rate is 1.5×10^{-11} cm^{3}s^{-1} calculate the PCE density of N_2^+

According to page 13 of the supplementary notes, dissociative recombination obeys the loss formula:: $L(N_e) = \kappa[e^-][XY^+]$. Here, κ i the rate coefficient.. This is the driver for our case, which is dominated by the charge exchange process. Continuing, on page 15 it is noted that $L(N_e) = \beta(h)\, N_e$; where $\beta(h) = \kappa[YZ]$.

Here: $L(N_e) = \kappa[O_2]N_e$ The equilibrium condition is that

$$q = L(N_e) \Rightarrow \gamma\, I\, n_{N_2}\, \sigma_{\text{ionization}} = \kappa[O_2]N_e$$

$$q = L(N_e) \Rightarrow 26.7\,\gamma\,\text{ions}/_{\text{cm}^3\text{ s}} = \left(1.5\times10^{-11}\ \text{cm}^3\text{ s}^{-1}\right)\left(5 \times 10^{7}\ \text{cm}^{-3}\right)N_e$$

$$26.7\,\gamma\,\text{ions}/_{\text{cm}^3\text{ s}} = \left(7.5\times10^{-4}\ \text{s}^{-1}\right)N_e$$

$$N_e = \frac{26.7}{7.5\times10^{-4}}\,\gamma\,\text{electrons}/_{\text{cm}^3} = 3.55\times10^{4}\ \text{electrons}/_{\text{cm}^3}$$